Fractal Voyager: A Web Application for Exploring and Studying Complex Dynamics

Dakota Bryan

ST. LAWRENCE UNIVERSITY

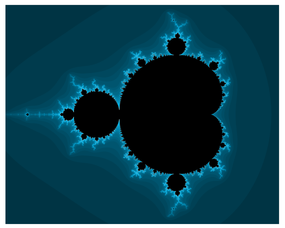
Complex Dynamics

- Complex Dynamics is a field of mathematics which studies the behavior of iterated functions in the complex plane
- The complex plane is the set of all numbers that have a real part and an imaginary part, so they can include the imaginary unit i, which is $\sqrt{-1}$
- Complex numbers are graphed with their real part on the horizontal axis, and their imaginary part of the vertical axis

The Parameter Plane

$$z \mapsto z^2 + c$$

- The parameter plane is the complex plane when a function is iterated until a condition is met with respect to a fixed point, in this case z, which is starts at $0 + 0i$
- It includes the parameter, c, which changes with respect to the passed in point across the complex plane

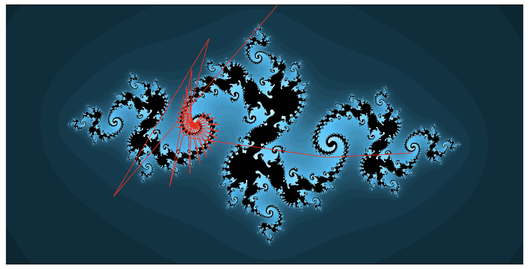


Parameter plane from $-2 \leq \Re \leq 1$, $-1.2 \leq \Im \leq 1.2$ for above function until z escapes to infinity. c is set the the complex number equivalent to each pixel

The Dynamical Plane and Orbits

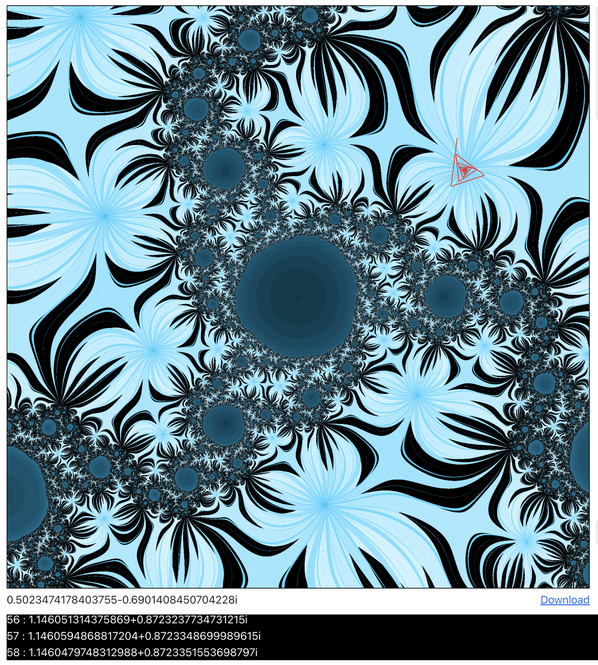
$$z \mapsto z^2 - 0.79 + 0.15i$$

- The dynamical plane is the complex plane when a function is iterated until a condition is met with respect to a variable point, z, that changes with respect to the passed in point across the complex plane
- A complex number's orbit for a particular dynamical system is the value of the number after each iteration of the function.



Dynamical plane from $-2 \leq \Re \leq 1$, $-1 \leq \Im \leq 1$ for above function until z escapes when z is set to numbers across the complex plane. Orbit for $1.14 - 0.14i$

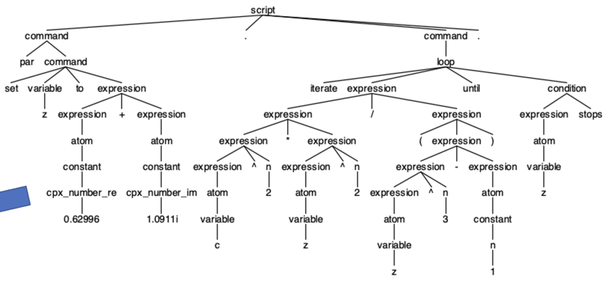
Application User Interface



Custom Language Compilation

Fractal Voyager uses a complex dynamics scripting language with a grammar defined with ANTLR. The script gets passed to ANTLR which creates a parse tree which is traversed to generate c++ code that gets compiled to Web Assembly in the browser. This code is ran on the complex number representation of each pixel to determine the coloring of that pixel.

```
par set z to 0.62996 + 1.0911i. iterate
c^2*z^2/(z^3-1) until z stops.
```




```
int calcPixel(double z_re, double z_im, double c_re, double c_im, int maxIters,
double minRadius, double maxRadius, int type, double epsilon) {
  std::complex<double> z(z_re, z_im);
  std::complex<double> c(c_re,c_im);
  if (type == 0) {
    z = std::complex<double>(0.62996,0)+std::complex<double>(0,1.0911);
  }
  std::complex<double>prev(z);
  for(int i = 1; i < maxIters; i++) {
    z = c*c*z*z/(z*z*z-std::complex<double>(1,0));
    if(abs(abs(prev) - abs( z)) < epsilon) {
      return i;
    }
    prev = z;
  }
  return 0;
}
```

```
expression: (PLUS | MINUS)? atom #SIGNED_ATOM_EXP |
            expression POW n #POW_EXP |
            expression TIMES expression #TIMES_EXP |
            expression DIVIDE expression #DIVIDE_EXP |
            left=expression PLUS right=expression #PLUS_EXP |
            expression MINUS expression #MINUS_EXP |
            cpx_function LPAREN expression RPAREN #CPX_FCN_EXP|
            LPAREN expression RPAREN #PAREN_EXP
            ;
```

HTML Canvas & Fractal Drawing

- The HTML canvas is an HTML element that can display image data and draw lines and boxes on the web
- To draw the fractal images, an array of image data is created and passed to a canvas element which consists of four elements for each pixel on the canvas, one for the red color intensity, one for blue, green, and one for transparency
- The color of a pixel is black if the condition is never met, or along a user-defined color gradient based on how quickly the condition is met

Web Assembly

- Web Assembly is binary code which can be executed on the web to make computationally intensive tasks much faster than the default language of the web, JavaScript
- This application uses Web Assembly, or WASM, in two distinct places
  1. The custom compiler which turns scripts into c++ code is compiled to WASM with emscripten (a compiler toolchain), and called from JavaScript
  2. The c++ code generated is compiled to WASM with a tool called emception (a verson of emscripten that is compiled with WASM), allowing this on-the-fly c++ code to be compiled and executable on the web

React

- React is a JavaScript library that this application is built with
- It allows applications to be built with reusable components, for example, the color gradient in the lower right of the app is a component which is passed the base color and how many colors to create, then that component renders this shown color box based on JavaScript code
- The state of components is stored with React, for example, the wasm c++ code to generate the fractal image data takes many parameters that are based on the options shown on screen. When one of the options changes based on user input, that triggers a state change, which allows for the update button to be clicked, which changes state variables that get passed to the c++ code to generate a new fractal.

Acknowledgements

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