

		6		9	2			7	8	6	2	1	7	3	4	5	9		9		5	6		3	7	9	4	1	5	6				
						6	8	2	5	3	4	6	8	9	1	2	7	3		5	7	8	2		6	4	3	2	7	8	1			
		4		7			1	9	1	9	7	2	5	4	8	6	3					9	1	5	8	2	5	6	8	2	7			
			9			3			6	8	2	7	1	3	6	5	9	8	4	5	7	4	6	7		9	4			2	8	9	1	3
			1	4			6		2	7	1	3	6	5	9	8	4	5	7	4	6	7					7	3	4	6	2			
	5		8	2					9	4	8	1	7	6	3	2	7	4	1	8	5	3	6	2	9	1	5	6	4	9				
3	2		7						9	1	7	1	9	5	3	8	2	4	6	9	5	8	4		1	4	9	3	7	5				
	9				1				3	2	8	4	9	6	5	7	1		7				8		3	6	7	2	3	8				
1			2			9			4	5	6	7	2	1	3	9	8					9	8		6	8	1	5	9	4				

Sudoku Graphs

Visualizing, Playing, and Solving

5	1	2	9	3		8		2					6	6	9	3	8	7	2	4	5	1	2				5			7	
9	3	7	8	6								5		5	8	1	6	9	4	3	7	2							6	1	
6	2	1	4	5				1				9		2	4	7	3	5	1	9	6	8		4	6		7	8			
7	6	8	2	4						2	3	8		1	9	5	7	3	8	2	9	4		7							
3	4	9	6	1			1		7					8	7	2	1	4	9	6	3	5					9	3	7	4	
2	8	3	5	7	5	6								4	3	9	5	2	6	8	1	7		9	1			5			
4	9	5	3	8		1	9					7		9	1	8	4	6	5	7	2	3			9		3	4		8	
1	5	4	7	2	7						4			3	6	4	2	1	7	5	8	9				9					2
8	7	6	1	9		2	8	9						7	2	5	9	8	3	1	4	6					2		4	6	

Cody Bryan

What is Sudoku?

- Logic puzzle
- 9 by 9 grid separated into 9 sections of 3 by 3 sub-grids
- Place the numbers 1 through 9 in the grid such that no number is repeated in a row, column, or sub grid
- Exactly one solution to a valid sudoku

4		5				9		1
8				9	1			
				2	7		4	
	4	8	6				3	
	3						6	9
	1	6				8		4
3		1	8		4	2		
						7	1	5
	9		2					

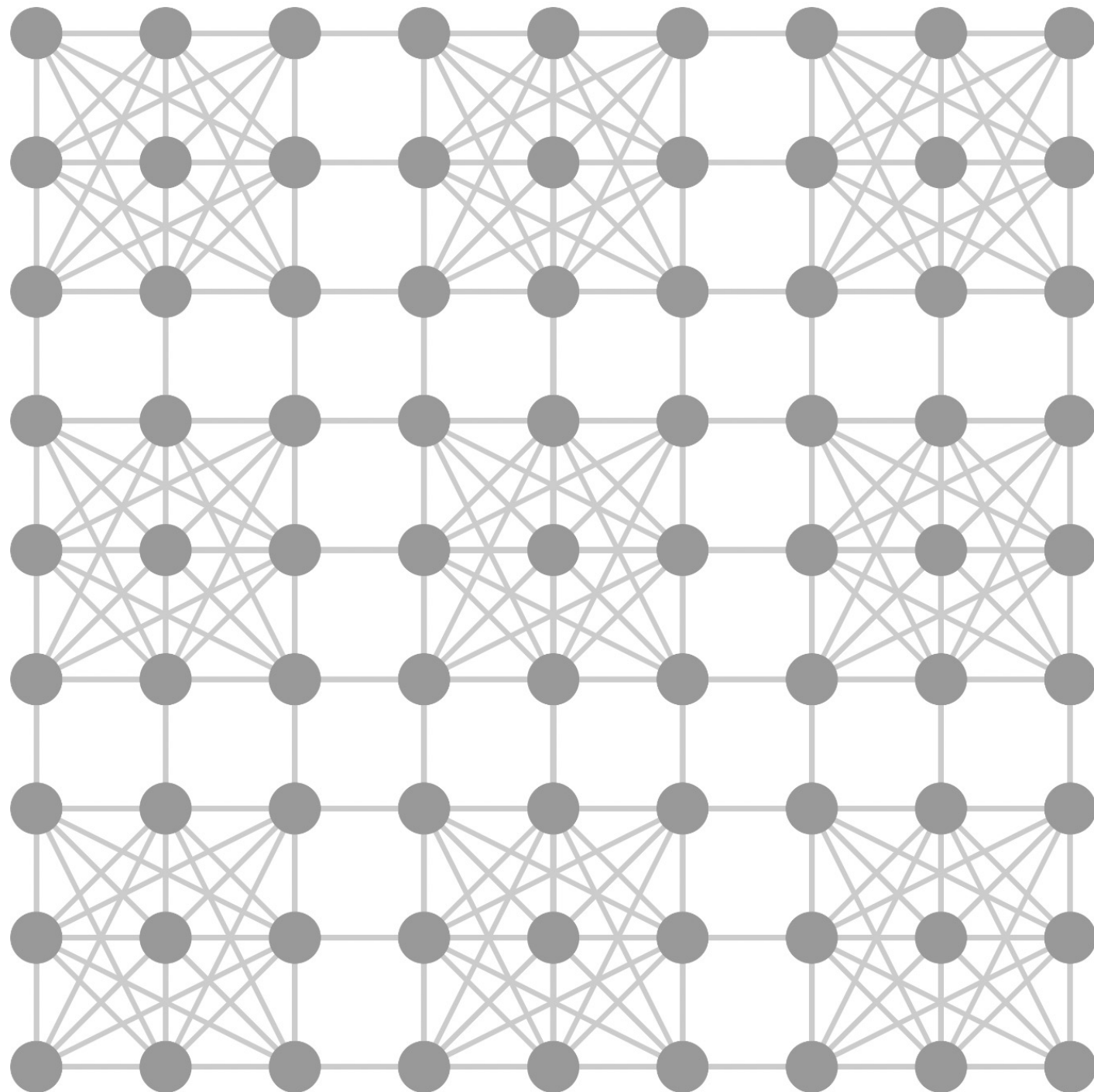
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4	2	5	3	8	6	9	7	1
8	7	3	4	9	1	3	2	2
1	6	9	5	2	7	3	4	8
7	4	8	6	1	2	1	3	7
5	3	2	1	4	8	1	6	9
9	1	6	7	5	3	8	2	4
3	5	1	8	7	4	2	9	6
2	8	4	3	6	9	7	1	5
6	9	7	2	1	5	4	8	3

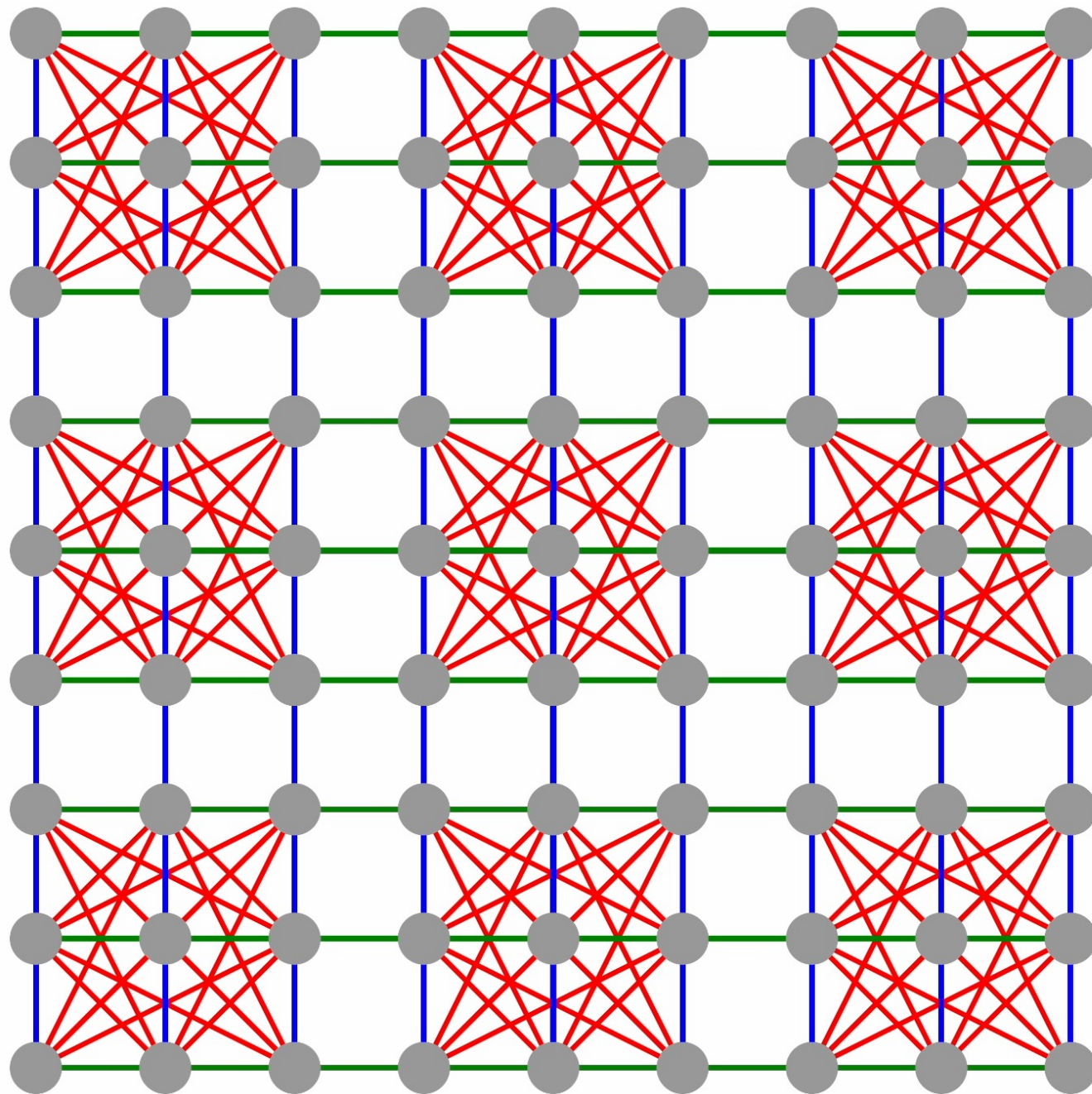
Sudoku Graph

- Vertices represent cells
- Adjacent vertices can't be same symbol
- 81 vertices, 20-regular, 810 edges
- Clique number is 9, each row, column, and block is a 9-vertex clique



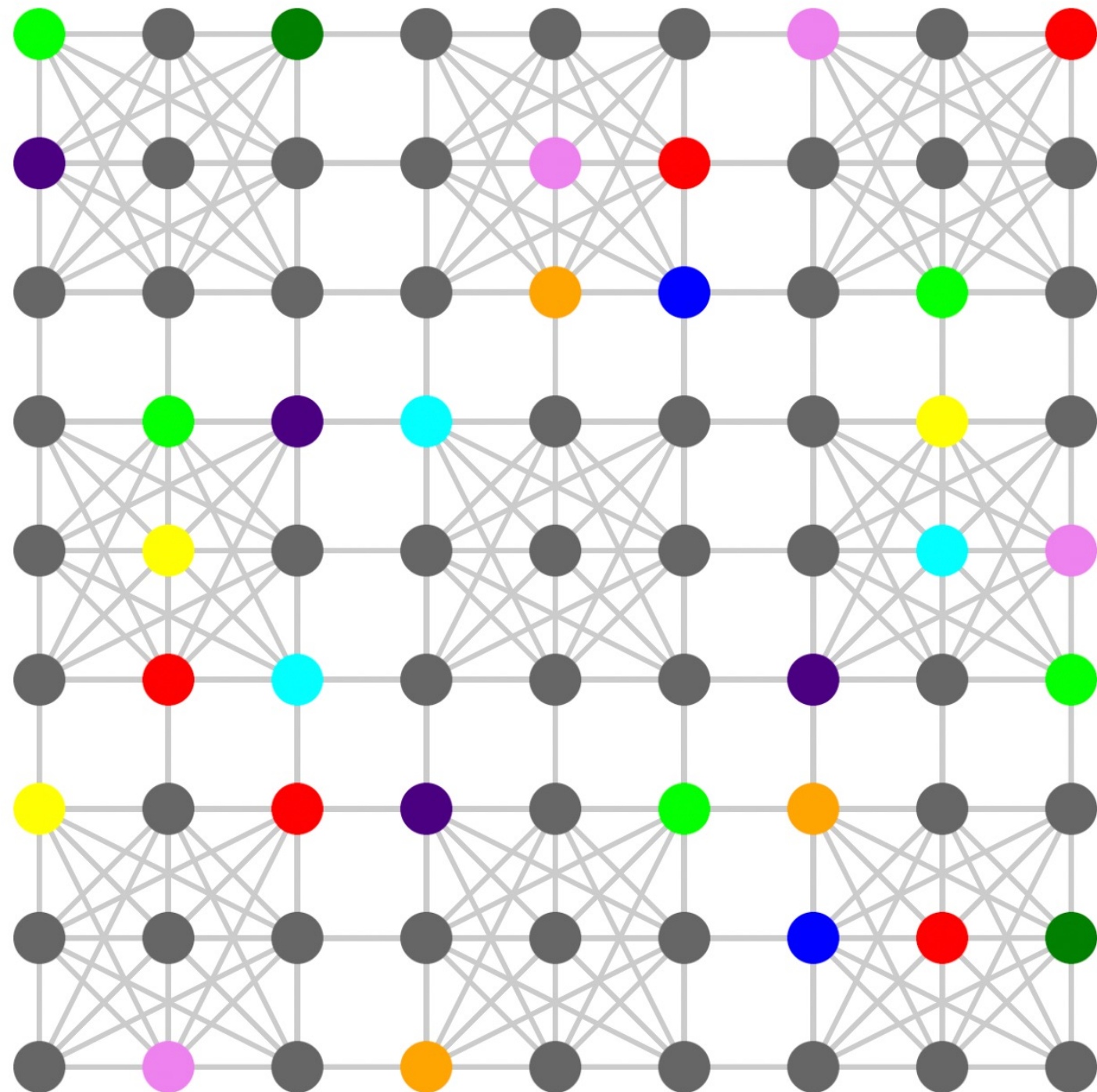
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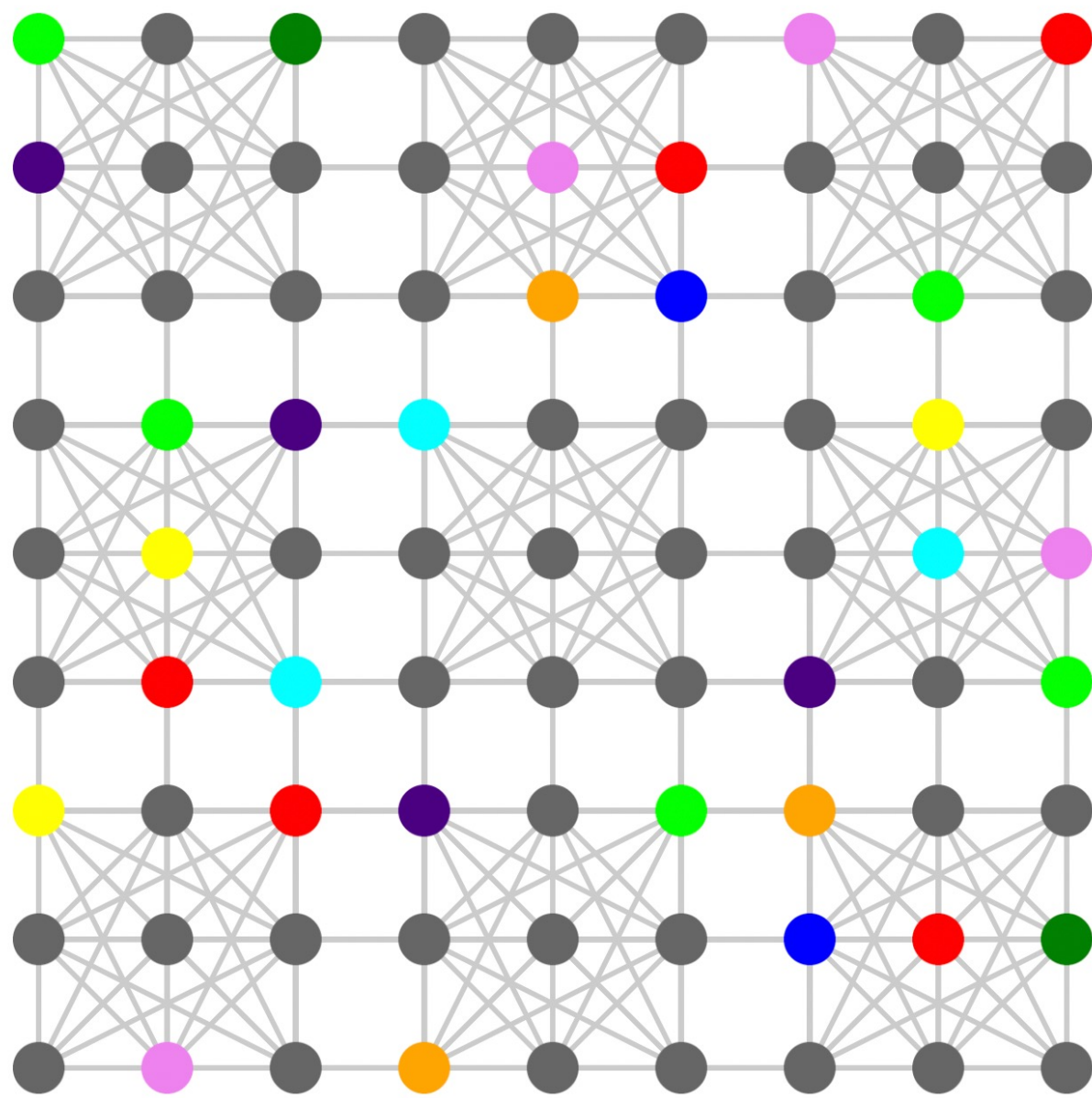


Sudoku Graph Coloring

- Numbers in grid translates to colors in graph
- If a given valid sudoku, graph has coloring number of 9, with one possible 9-coloring



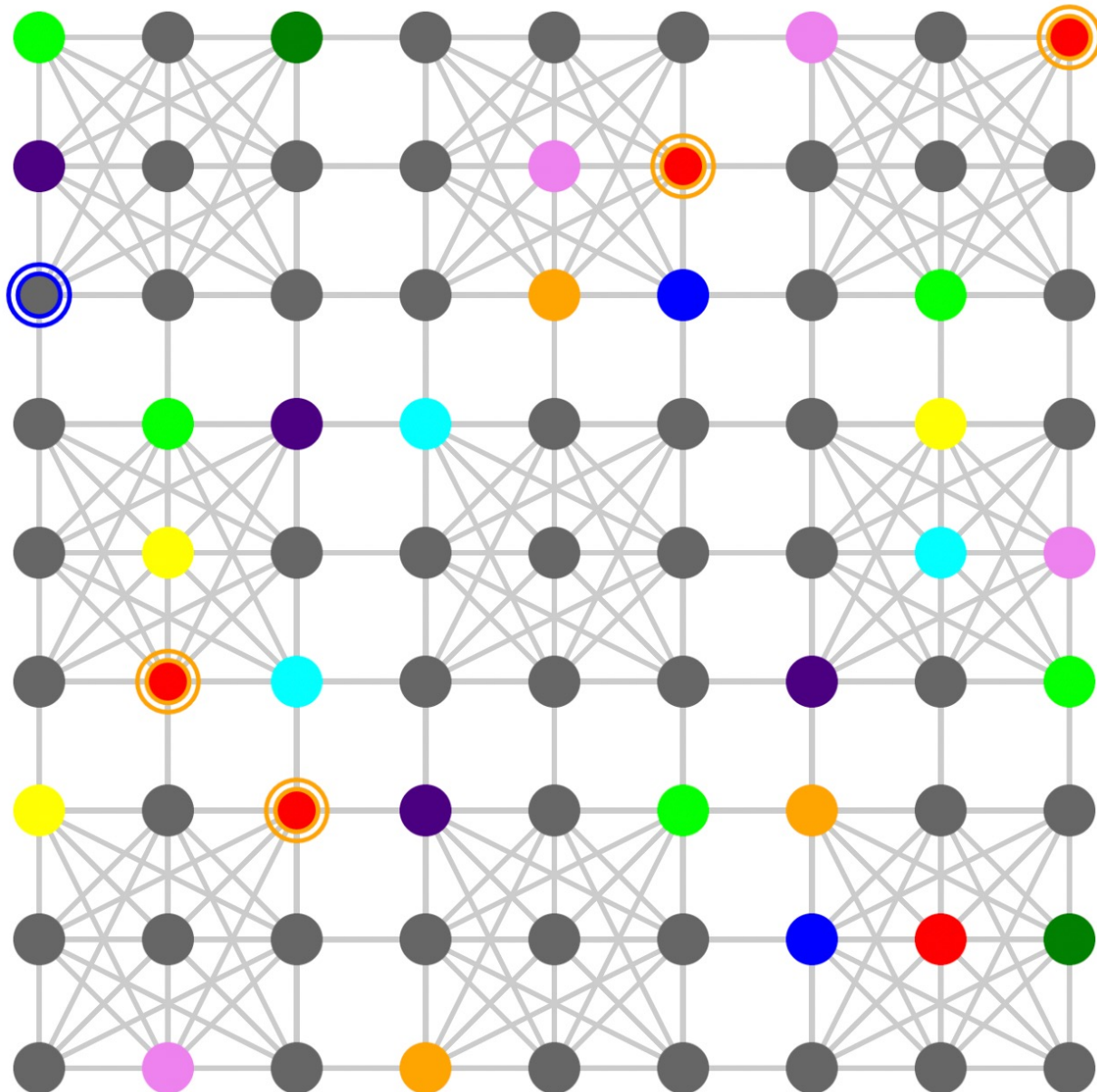
4		5				9		1
8				9	1			
				2	7		4	
	4	8	6				3	
	3						6	9
	1	6				8		4
3		1	8		4	2		
						7	1	5
	9		2					



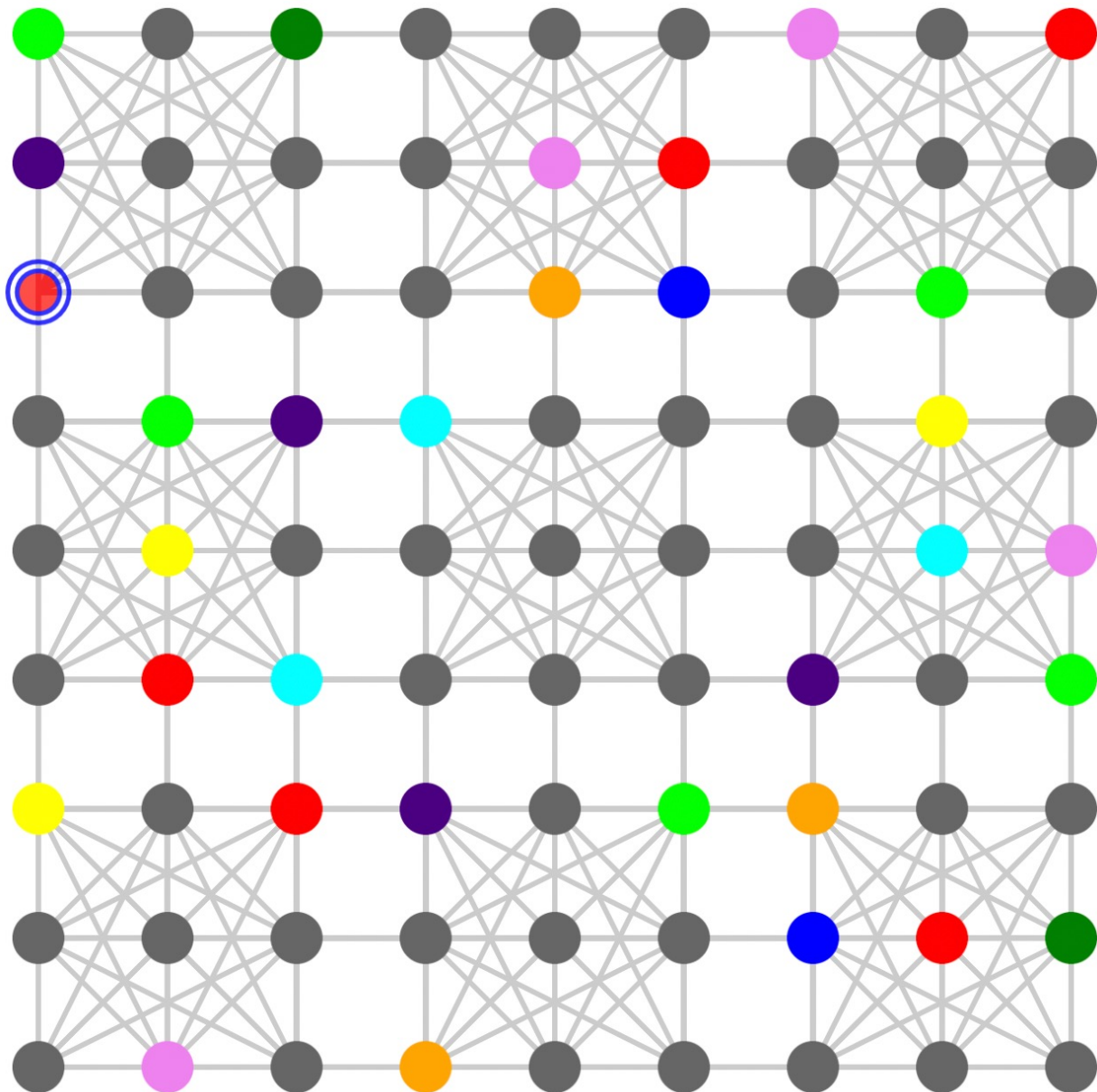
Basic Solving Strategies

Only option in clique, single candidate

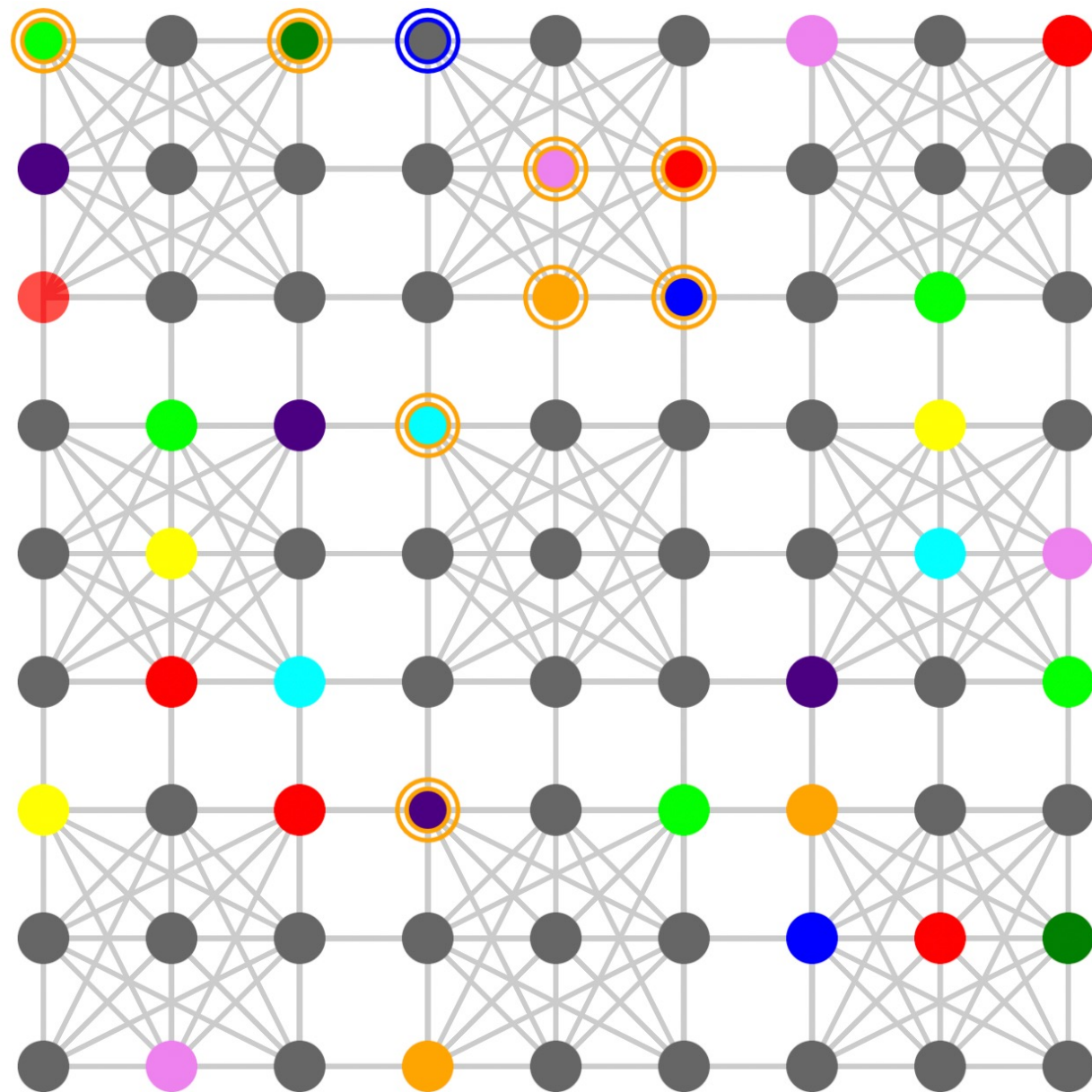
4		5				9		1
8				9	1			
				2	7		4	
	4	8	6				3	
	3						6	9
	1	6				8		4
3		1	8		4	2		
						7	1	5
	9		2					



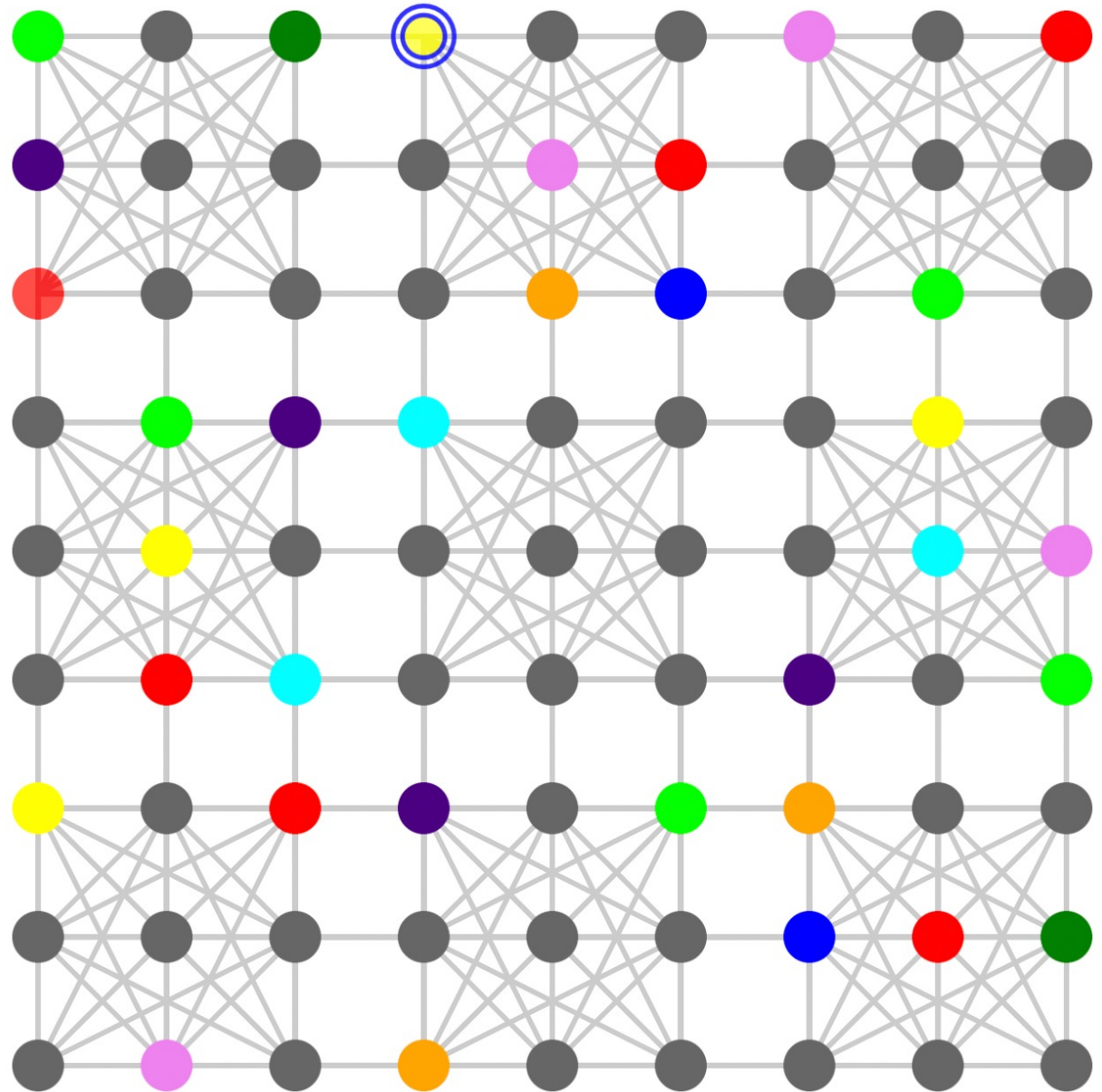
4		5				9		1
8				9	1			
1				2	7		4	
	4	8	6				3	
	3						6	9
	1	6				8		4
3		1	8		4	2		
						7	1	5
	9		2					



4		5			9		1	
8				9	1			
1				2	7		4	
	4	8	6				3	
	3						6	9
	1	6				8		4
3		1	8		4	2		
						7	1	5
	9		2					



4		5	3			9		1
8				9	1			
1				2	7		4	
	4	8	6				3	
	3						6	9
	1	6				8		4
3		1	8		4	2		
						7	1	5
	9		2					



Algorithms

Backtracking graph coloring, board
generation

Backtracking Coloring

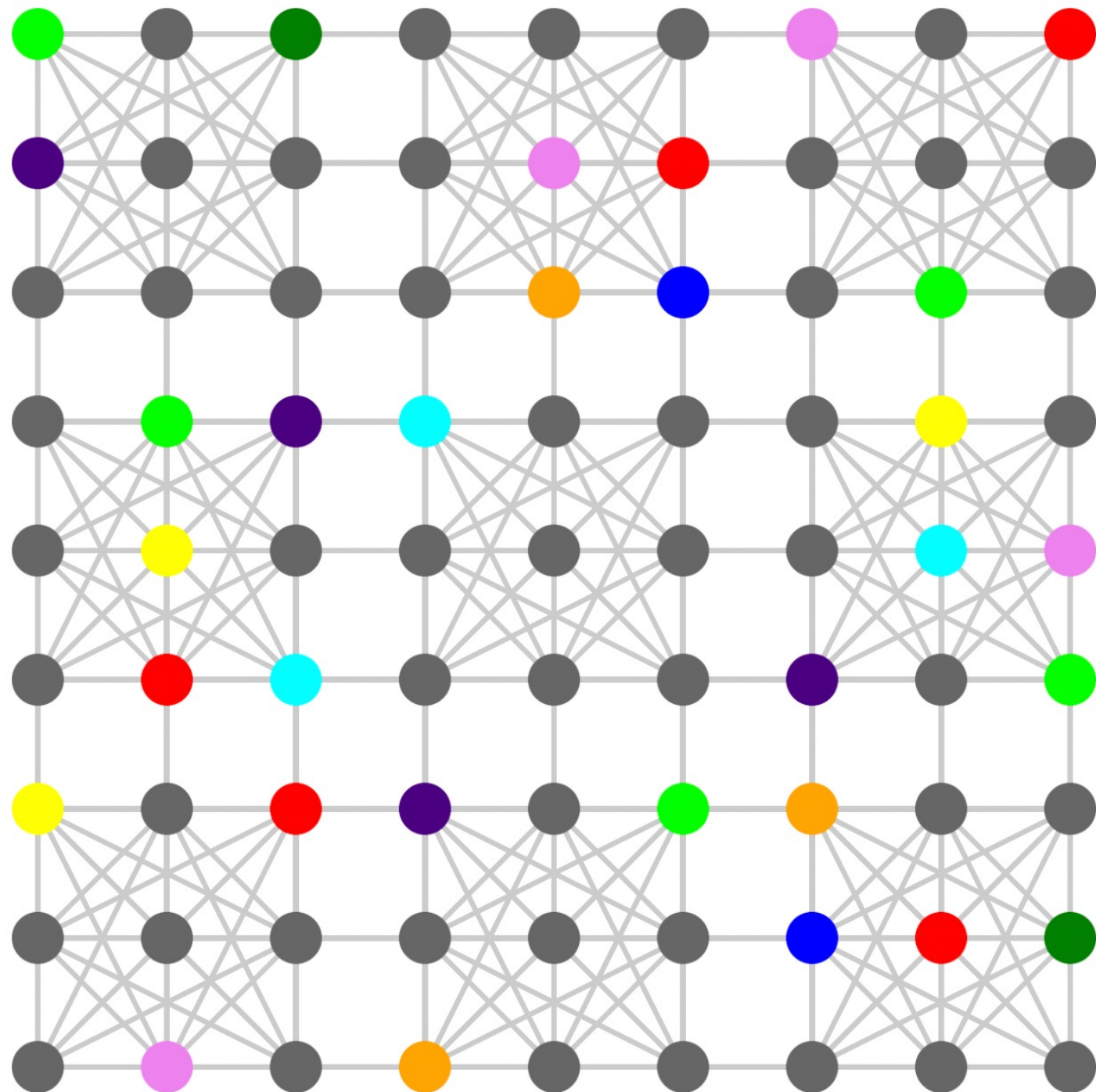
1. Loop through all uncolored vertices, selecting one each time

2. Loop through possible colors for that vertex, coloring it that color each time

Repeat steps 1-2

If there are no possible colors for a vertex, or all options have been tried - go back a step (uncolor)

If all vertices have been colored - check if it is correct - if so, correct solution, if not, incorrect start



Board Generation

A. Backtracking algorithm to generate a complete, random board from empty board

B. Backtracking algorithm to “delete” cells until there is a certain number left

1. Delete a single cell

2. Run a modified backtracking solving algorithm that counts number of solutions for given board

3. If step 2 yields more than 1 solution, go back a step.

4. Repeat 1 through 3 with the new board until there is the given certain numbers of left

4		5				9		1
8				9	1			
				2	7		4	
	4	8	6				3	
	3						6	9
	1	6				8		4
3		1	8		4	2		
						7	1	5
	9		2					

Demo

<https://dbryan17.github.io/sudokuGraph>